The social structure of the Spartan City-State: A game-theoretic explanation *Jiaxin Liu*

Abstract: This paper seeks to explain Archaic and Classical Spartan society through game-theoretic measures. Namely, it argues that the strict hierarchy and relative stability of the *Lacedaemon polis* (Spartan city-state) is the result of a game equilibrium that both the elite (*Spartiates*) and the non-elite (*Helots*) of society—who are modeled as independent "game players"—are unwilling to deviate from. Yet, it is also emphasized how such an equilibrium could in fact be disrupted, and how the Spartiates and Helots were incentivized to not maintain or maintain the status quo under certain conditions. These disruptions were manifested through socio-economic inequalities among the Spartiates and Helot rebellions. Exogenous factors such as natural disasters and intervention by foreign city-states are also considered in the analysis since they influence the construction of Spartan society as well. Overall, this study highlights the possibility of a quantitative study of historical class tensions, and provides a unique perspective on one of the most distinct and well-known polities of antiquity.

Keywords: Sparta; Ancient Greece; Game Theory; Analytic Narratives; Social History; Class Conflict

Alongside the city state of Athens, Classical Sparta has not only attained much renown throughout history, but in contemporaneity as well. The enduring myth of its austere, militaristic character finds its reflection in quotidian realities: the movies 300 and *Troy*, video-game series such as *Assassin's Creed* and *God of War*, as well as vocabularies such as "laconic" and "spartan" (Cartledge, 2004; Cole, 2019). Its popularity in the collective imagination is not unique: Laconophilia, or an admiration of Spartan culture, is a prevalent phenomenon throughout history, stretching from Platonists of antiquity to Modern Revolutionaries such as Rousseau and the Zionist movement in Israel (Shapiro, 1974; Rawson, 1969). Its wide appeal to audiences so diverse point to the layered nature of Spartan significance. On one level, the Spartan Hoplites are seen to be embodiments of unbridled masculinity and discipline; on another level, the Spartan constitution, noteworthy for its hierarchy, xenophobia, and tradition, inspired political regimes that followed its conservative footsteps (Cartledge, 2004). Its relevance to the human experience justifies our research into deciphering its social structure.

Spartan society is remarkably distinct from contemporaneous city-states in Greece, or even history, for that matter (Cartledge, 2013). Defined by an excessive bent on militarization, tradition, socialism, and hierarchy, it stood in stark contrast to the conventional picture of mercantile city-states of Greece, best exemplified by Corinth and Athens.

Sparta's constitution, which laid the groundwork for its cultural idiosyncrasies, was attributed to an apocryphal figure —Lycurgus—who proclaimed the essential Spartan virtues of equality, military fitness, and austerity (Cartledge, 2013). The constitution spelled out Spartan

practices that upheld those virtues. All young boys within the city were forced to undergo initiatory military training in the agoge, while young girls were similarly enrolled in a state-wide curriculum for education (Cartledge, 2013; Pomeroy, 2002). The emphasis on martial lifestyle was complemented by artificial arrangements of citizen-life. Males had to attend the syssitia, a common mess where individuals-regardless of wealth and status-bonded together and developed military brotherhood by bringing agricultural produce from their plots of land, kleroi (Hodkinson, 2009). Precious metals were patently forbidden, and any inappropriate displays of wealth were punishable—an emphatic opinion on socio-economic equality. Fairness was further established along the lines of sex: women were allowed greater freedom than other city-states as they were seen to be mothers of the fighting force, and could inherit massive properties of land (Hodkinson, 2002). Crucially, the state was paternalistic, interfering heavily in the citizens' private lives. Marriage customs were ritualized, and a whole set of punishment-encouragement mechanisms ensured healthy offspring (Pomeroy, 2002). To avoid sentimental attachment to the household, men under 30 were forbidden to cohabit with their wives, and occasional home visits were shrouded in secrecy—an unwritten procedure (Cartledge, 1981). These draconian measures ensured combat-readiness of men by preventing social detachment from state duties. Strong cohesive and cooperative measures defined Spartan state institutions.

It is therefore incredulous that such massive institutions were miraculously established by just one legislator, whose existence is embroiled in inconsistencies and myth. A more adequate explanation was sought. Both Plutarch Lycurgus 28 and Thucydides 4.80 asserted that "Spartan institutions have always been designed with a view to security" (Thuc., Warner Translation). The unique demographics of Sparta—specifically, the ever-present threat of internal revolt justified its paranoia-like focus on militarization. Sparta's society was categorized into *Spartiates*, Perioikoi, and the Helots. The Spartiates, or Homoioi (equals), were citizens proper, and resided in the city of Sparta. These individuals were the ones who were subjected to the aforementioned Spartan practices (Hodkinson, 2009). The Perioikoi, or city-dwellers, resided in small townships along the countryside. Unlike the Spartiates, they were not required to undergo military training and not subjected to the restrictions that we have associated the Spartiates with, but were free subjects whose main role were to engage in commerce and manufacture, activities forbidden to the Spartiates (Cartledge, 2013). They maintained the bare minimum of what was a primitive Spartan economy. These classes owned Helots, ethnic Greeks mostly from conquered Messenia, a neighboring province, who formed the servile population. Concurrently, these Helots were subjects of the state, and according to new studies, most likely had a social standing analogous to serfs in Medieval Europe. They produced agriculture by working on their masters' kleroi (Hodkinson, 2009).

Antagonisms between the Spartiates and the Helots were extremely pronounced, and history is replete with occasions of rebellions and the ensuing repressions, most startlingly displayed in the annual, ceremonial declarations of war by the ephors (*overseers*) against the Helots to affirm their superiority. Upon announcement, a selected band of young men (*krypteia*) were sent to indiscriminately kill any Helot they encountered in the countryside (Hodkinson, 2009). The Spartiates maltreated the Helots routinely—they "imposed on the Helots every kind of insulting work which led to total degradation", as attested by Myron of Priene, a Spartan historian (Plut. *Lycurgus* 28, H. Clough translation). Oftentimes, they would forcefully inebriate the Helots, thereby publicly shaming their drunken actions in the *syssitia*. Disaffection was further illustrated in annual beatings to remind them their place. While these accounts were

often embellished and exaggerated, scholars have agreed upon the simmering tensions between the two classes. Critias, in his fragmentary *Spartan Constitution* mentioned the Spartiate tradition of removing their spears only when they reached home and have locked all doors (Diels, Critias 27). Aristotle also emphasized the Spartiates' fearful attitude towards the Helots, implying great hostility between the classes and the possibility of conflict (Aristotle, *Politics* 1269a). Perhaps this sentiment was most suitably expressed by Paul Cartledge, who pointed that "the history of Sparta ... is fundamentally the history of the class struggle between the Spartans and the Helots" (Cartledge, 1987). Therefore, the social milieu seemed to be a suitable motivator for the distinct customs and legislature of Sparta. In this light, the practices of cruelty appeared to be symbolic mechanisms of power used to maintain the status quo.

Literature Review

However, most conventional literature suffers from low-resolution explanations. While security could always be at the back of the Spartiates' heads, why was it that they were willing to go to such depths to deter the internal threat? The militarization came at the cost of a bare-bones economy and greatly curtailed freedom for socio-economic opportunities. Additionally, why was it that the Helots, under normal circumstances, refrained from revolting if the Spartiates engaged in such extreme measures of oppression? A more precise explanation is required, and I attempt to do so via mathematical modeling. Analytical approaches to history are not new, and I mainly take inspiration from the eponymous work *Analytic Narratives*, a compendium of essays written by Robert H. Bates, Avner Greif, and Margaret Levi (Bates, Greif, Levi, & Rosenthal, 1998). The theoretical framework comes from Avinash Dixit's *Games of Strategy* (Dixit, Skeath, & Reiley, 2015).

Game theory can be defined as the study of mathematical models of conflict and cooperation between intelligent, rational decision-makers (Myerson, 1997). A game is any social situation which involves more than two players. Game theory assumes that these players are autonomous and rational. This means that the players have their own preferences and act in their own self-interest. It also means that they will act so as to maximize their self-interest. A distinct characteristic of game theory is that the players' decisions are interdependent. Players consider their moves in light of what they think the other player's decisions are, and vice versa. To demonstrate with an example, we suppose that players A and B are competing restaurant chains. Player A can choose to increase or decrease the price of its food. Player B has the same two options. Player A's moves—whether to increase or decrease its food's price— are evaluated on the basis of player B's decision to increase or decrease its food price, and vice versa. This is a simple, competitive game theory model with 4 outcomes in total, and players A and B evaluate and choose the decision that will lead to the best outcome for them. In some cases, a Nash Equilibrium is met. This means that neither A nor B will want to deviate from an outcome, as alternative outcomes will yield inferior results. No matter what the other player has in mind, the Nash Equilibrium is the rational outcome that any self-interested player will gravitate towards.

In the following analysis, I shall illustrate how the class struggle between the Helots and the Spartiates eventually settled into an equilibrium of Spartiate cooperation and Helot oppression

which characterized their historical relationship. Yet, I also highlight how the equilibrium could be disrupted, being occasionally revealed as Helot rebellions and internal strife amongst Spartiates. Crucially, we decipher what factors lead to the decision of *rebellion* amongst the Helots or *cooperation* amongst the Spartiates. A mathematical model allows for a more comprehensive picture of historical contingencies—what could have happened, why did it not happen, and what adjustments would have incurred those possibilities. Fundamentally, such an approach dispels the Spartan mirage—an irrational idealization of Spartan exceptionality as a "riddle wrapped in a mystery in an enigma"—by demonstrating how Spartan society was structured the way it was not because of some cultural proclivity to military communism but existing social conditions (Cartledge, 2018).¹

Model

To construct a model, several important assumptions need to be made.

Firstly, this attempt is not a be-all-end-all description of the Spartan society. Reality is often complex, with many uncertainties, factors, and players. Therefore, the explanatory power of a model is greatly limited. Nonetheless, this attempt serves to illustrate a new perspective to approaching history, and by doing so, sheds light on factors that were previously ignored.

Secondly, the model shall be premised on Cartledge's pithy summarization of Spartan history as a class struggle between Helots and Spartiates. Therefore, we have two players: Spartiates and Helots. Each of them has two decisions: cooperate/don't cooperate and rebel/don't rebel respectively. They are playing a game that defines the social structure of the Spartan state. Here, cooperation means achieving the Lycurgan ideal-an authoritarian, militaristic regime which demanded uniformity among all eligible Spartiates. This was not necessarily the case in reality. Even though the Lycurgan constitution seemed to characterize Spartan society, recent scholars discerned signs of socio-economic inequalities amongst the Spartiate population which belied official rhetoric (Hodkinson, 2009). Hoarding of precious metals, land ownership, and expensive surpluses in the syssitia were some indications of well-off Spartiates who dominated leadership positions (Ober & Weingast, 2016). On the other hand, urban archaeological research demonstrated that the poor lived on meager subsistence, potentially having to seek assistance from wealthier citizens (Hodkinson, 2009). More directly, Tyrtaios mentioned that the occasional civil war happened when "some are excessively poor and others excessively rich", while Aristotle pointed to the concrete instance of the Second Messenian War when "certain men ... demanded a redistribution of territory" (Aristotle, Politics Pol. 1306b36-1307a2; Hodkinson, 2009).

The ostensible uniformity demanded by the Spartan constitution and the barracks lifestyle likely incurs an opportunity cost for richer Spartiates, for they could have utilised their resources into furthering their wealth, be it through commerce with other city-states, monopolizing farmland, or engaging in mass production of specialized goods (Hodkinson, 2009). The cloistered, economically unproductive lifestyle of the citizen-soldier precludes these opportunities. Conversely, poorer Spartiates do not suffer significant cost from cooperation.

Rather, they potentially benefit from it, since the military lifestyle ensures lodging, sustenance, and security. The risk of being exploited by the hands of richer Spartiates greatly decreases once the "wealthiest Spartiate in principle consumed at a level determined by the poorest of his fellow Spartiates" at the military phitidia (dining associations) (Ober & Weingast, 2016, 6). Equality guaranteed from the military lifestyle increases the likelihood that even the poorest Spartiates can contribute their agricultural produce to the syssitia, a requirement of being a citizen (Hodkinson, 2009). The consequences of not contributing were severe: known as hypomeiones-the inferiors-these ex-Spartiates were stripped of privileges and occupied the underclass that was slightly above the Helots in social standing. To worsen matters, the shame inflicted was hereditary, indicating the permanent, irreversible downgrade of status (Hawkins, 2011). Therefore, there is benefit of cooperation. Yet, this does not mean that only the poor benefit and the rich suffer from cooperation. Effective citizen-soldiers are a strong deterrent against rebellion by the large numbers of slave-farmers owned by wealthier Spartiates. Similarly, the constant state interference into personal autonomy and the immense societal pressure of militarization-exemplified by the usage of social and corporeal punishments to reinforce Spartiate core values of equality, fitness, and austerity—are likely unconducive for the mental wellness of all Spartiates regardless of their socio-economics statuses (Knottnerus & Perry, 2002). We conclude that the net payoff of cooperation is an aggregate and ambiguous value due to its varying effects on different sectors of the population.

Thirdly, other factors relevant to the contribution of the equilibrium—intervention by foreign city-states and natural disasters—are not treated within the model as they do not immediately affect the outcome, which is the result of independent actions by the Spartiates and Helots. Nonetheless, they will be considered in an evaluation of how the equilibrium might possibly change with the inclusion of these possibilities. For example, an increase in external threat by a malicious foreign power is likely to increase the likelihood of Helots to rebel, for they can collude for an increased possibility of success. This has happened in the Peloponnesian War when the Athenians came to the assistance of the Helots (Kagan, 1987).

We can raise 4 possibilities with the 2 actions each for Helots and Spartiates. Respectively, they are [*Rebel, Cooperate*], [*Rebel, Don't Cooperate*], [*Don't Rebel, Cooperate*], and [*Don't Rebel, Don't Cooperate*].

Case 1: [Rebel, Cooperate]

This happens when the Spartiates decide to abide by the Spartan constitution and the military lifestyle, and the Helots decide to revolt. The two classes engage in direct conflict. Due to lengthy preparations for such a possibility, it is likely that the Spartiates will win. However, the victory will be costly: whilst there is no consensual answer to the ratio of Helot to Spartiate populations, it is widely agreed to be around 1:9 or 1:10, or for conservative estimates, 1:7 (Kagan, 1987). Regardless of their famed fighting prowess, the sheer quantity of the slaves will inflict heavy relative losses to the Spartiate army. Moreover, it is likely that the Spartiate army—in order to justify their sunk costs—will pursue an absolute victory. The result of winning the war, to the Spartiates, will be renewed exploitation of the Helots' production, likely to be more sustained,

exhaustive, and cruel after the mellowing down of rebellious intent and the revanchism of the emboldened victors.

Therefore, the payoff of *Case 1* for Spartiates entails the algebraic summation of the following factors. Firstly, there is a net payoff for cooperation, as analyzed above. Secondly, there is the cost of manpower in defeating the Helots. Thirdly, there is the benefit of exploiting the Helots after they have been subjugated. As argued above, such exploitation is likely to be intensified, serving the utility as an additional punishment that the Helots deserve for disregarding their masters. For simplicity's sake, we assume the zero-sum nature of the game, and for the Helots, the payoff in *Case 1* will be the algebraic summation of two factors: their increased loss of welfare following their exploitation, and the immediate loss of manpower (i.e casualty rates) by losing against the Spartiates. It is important to note that given the assumption of the game's zero-sum nature, the numeric value of the Helots. This aids in our analysis later on.

Case 2: [Rebel, Don't Cooperate]

Case 2 happens when Spartiates refuse to cooperate, meaning that there will be significant internal class struggles within citizens-displayed in the form of socio-economic inequalities. This obstructs the opportunity to establish a cohesive and efficient military regime. We can explain this more clearly with the proportionality principle, developed by Cox, North, and Weingast (Cox & Weingast, 2018). Sparta is a "limited access order", meaning that access to state institutions and privileges were reserved for a small elite, the Spartiates. The elite therefore had the sole potential to utilize disruptive violence which threatened social fabric, and constructive violence which sustained their ascendancy over the Helots. In economic terms, each Spartiate would not use their disruptive potential if and only if they had received a proportionate share of economic rent. If not, cost-benefit analysis would tell them that utilizing disruptive violence or colluding with the Helots yielded a higher payoff. In Sparta's case, the instability of the Spartiates' dominance as reflected by the atmosphere of fear and contrived measures to suppress the underclass indicated the *equality* of disruptive potential. The difference in violence potential between the most and least powerful Spartiate paled in comparison to the looming danger of Helot rebellion (Ober & Weingast, 2016). Therefore, socio-economic inequalities likely disrupt the equilibrium, resulting in a descent to violence as internal discord occurs.

Concurrently, the Helots choose to rebel. Faced with a huge numerical disadvantage and internecine strife, the Spartiates will suffer a devastating defeat. Therefore, its payoff will be the algebraic summation of two values: casualty rates by fighting the Helots, and the loss of welfare following the Helots' exploitation of Spartiates as roles are reversed. Given the large Helot numbers and the disorganization of the Spartiates, it is likely for the Helots to overwhelm the Spartiates, incurring negligible losses. Additionally, the Helots were an indispensable part of the Spartan ecosystem: their agrarian role supplied cereal, wine, and olive oil for the Spartiates (Cartledge, 2003). A revolt will result in supply shortages, quickly withering the morale and capability to muster arms within the city. Therefore, the net payoff for the Helots will only be

constituted of their increase in welfare as they exploit the Spartiates, who have become their slaves.

Case 3: [Don't Rebel, Cooperate]

Case 3 refers to the scenario when the Helots do not rebel and the Spartiates cooperate. In this scenario, the Spartiates incur the payoff of cooperation, but also reap the benefit of exploitation as well. The Helots on the other hand, suffer exploitation by the hands of the Spartiates. Unlike *Case 1*, there is no rationale for increased exploitation as the Helots have not been and will not be engaging the Spartiates in conflict.

Case 4: [Don't Rebel, Don't Cooperate]

Case 4 happens when the Helots refuse to rebel and the Spartiates refuse to cooperate. In such a situation, payoffs for both players are mirror images of each other: the Spartiates benefit from exploitation, while the Helots suffer from a loss of welfare. While this might seem unlikely for the Helots to accept exploitation when the Spartiate military is disorganized and weak, it is justifiable for the Spartiates if the net payoff for cooperation is sufficiently low enough

Analysis

To solve this game, we notice that when Spartiates cooperate or don't cooperate, there is no strictly dominant strategy for Helots. When the Spartiates cooperate, Helots will not choose to rebel, for fighting against an unwinnable conflict is worse than accepting one's fate as serfs. Yet, when Spartiates do not choose to cooperate, the weak Spartiate core incentivizes Helots to rebel.

Concurrently, we analyze the Spartiates' decisions when the Helots rebel or don't rebel. When the Helots rebel, it is more advantageous for the Spartiates to cooperate. While the details for justifying this conclusion can only be explained mathematically, as attached in the appendix, this statement can also be intuitively understood. The main drawback of cooperation when the Helots rebel is its potential cost to society, as well as the loss of manpower by fighting a war. This still pales in comparison to the consequence of not cooperating: a severe drainage in Spartiate manpower due to their numerical disadvantage, as well as the exploitation by former slaves. Cooperation is probably the best strategy for Spartiates when the Helots rebel. When Helots do not choose to rebel, the options of cooperation and non-cooperation only differ in the potential payoff of cooperation. If the payoff is positive, it is in the Spartiates' interest to cooperate when Helots do not rebel. Therefore, it is a dominant strategy for Spartiates to cooperate no matter the helots' decision.

Knowing this, the Helots will not choose to rebel, as analyzed above. The reduced set of possible outcomes result in a Nash Equilibrium of *Case 3*, where the Spartiates cooperate and the Helots do not rebel. It is an outcome where neither party wants to deviate from. Therefore,

regardless of whether the Helots know about the Spartiates' desire to cooperate or not and vice versa, *Case 3* is the logical outcome for both players. This reflects the historical status quo of Spartan society: strong military cohesion indicated by Lycurgus' constitution and relatively long stretches of stability from the 7th to 4th century B.C (Cartledge, 2013). We can therefore explain how the hierarchy was in fact a rational social arrangement for the two classes.

Yet, if the net payoff of cooperation is negative, there is no pure-strategy equilibrium, for any outcome will have one player deviating in search for a better payoff. The pure-strategy equilibrium is conditional.

Social developments throughout Spartan history—declining Spartiate military prowess in the 4th century B.C and multiple Helot revolts—reveal how decisions to rebel or cooperate were periodically reviewed. The volatile outcomes affirm the uncertainty of the Nash Equilibrium by demonstrating many instances when the net payoff of cooperation is negative. One salient example was in late 4th and early 5th centuries B.C, when private ownership rules eclipsed the Lycurgan ideal of equitable land ownership reforms that were administered by the state. Land inheritance customs mandated partible inheritance amongst progeny (Hodkinson, 2009). This incentivized the rich to marry the rich to minimize losses in estate size, resulting in a decrease in the payoff for cooperation as some Spartiates benefited much more from upholding the status quo. Increasing socio-economic inequality led to internal dissonance and a weakened fighting force-this sentiment was reflected in a lament on the deterioration of Spartan moral fiber in 390 B.C: "If anyone asked me whether I think the laws of Lycurgus still today remain unshaken, I could no longer assert this with confidence (Cawkwell, 1983)." The game's outcome deviated to Case 4, where neither the Helots rebel nor the Spartiates cooperate. However, this was not in the Helots' interest, for they could have gotten a better payoff by rebelling. The outcome shifted again to Case 2, when the Spartiates remained disorganized but the Helots decided to revolt. This was attested by increasing social turmoil and class conflict in the 4th century before culminating in the Battle of Leuctra and the 3rd century social revolution (Hadas, 1932; Cartledge, 2013). The battle freed most of the Messenian Helots, while the revolution sought to restore Lycurgan austerity and hierarchy among Spartiates.

Evaluation

While we cannot attribute specific values to the variables, the mathematical analyses as depicted above illustrate a much better understanding of how factors affect the outcome of the equilibrium in Spartan society. The crucial factor—the net payoff of cooperation—determines the nature of the equilibrium. As evidenced by the constant tussle between the state narrative of equality and the ever-existing socio-economic differences, the net payoff of cooperation is indeterminate, indicating some degree of fragility in Sparta's social structure.

Yet, stability had always been an unspoken virtue sought after by the Spartiates, and they nominally succeeded in achieving it with their totalitarian measures. The Spartiates' military ability deterred rebellion, and their exploitation coerced the helots into obedience, a brute method which worked quite effectively. Similarly, we can suggest that the liminal existence of the Perioikoi seemed to be a mediating force that contrived a shift to a pure-strategy equilibrium. Their loyalty to the Spartiates and participation in military conflicts helped to compensate for

the Spartiates' numerical disadvantage, disincentivizing Helot rebellions (Silva, 2015). With the option of rebellion being strictly dominated, the reduced set of outcomes of *Case 3* and *Case 4* allowed for Spartiates superiority over the Helots in all possible equilibriums. Yet, revolts did occur here and there, revealing latent social tensions that have not been totally suppressed. The Messenian Wars, Pausanias' plot, and The First Peloponnesian War were testimonies to outcome-shifting in the payoff matrix to Helot rebellions, incurred by changes to any of the exogenous parameters.

Importantly, this model excludes other factors relevant to the equilibrium—intervention by foreign city-states, complex Spartiate-Helot relationships, and natural disasters.

City-State Alliances

Foreign city-states often had complex dynamics with the Spartan polis, which can affect the equilibrium. In a bid to secure external security, Sparta was the first major power to initiate the Peloponnesian League, a system of mutual alliances in the Peloponnesian peninsula. Yet, such alliances were Sparta-centric: league members were only bound to Spartans, and had the same friends and enemies as Sparta had (Larsen, 1993). L. H. Jeffery summarized it as "a circle centered on Sparta, with the spokes of a wheel but not necessarily with the added cross-links of a web (Jeffery, 1988)." Therefore, we see noticeable efforts in trying to establish as much external support as possible without incurring too much cost. Doing so allowed the Spartiates to afford non-cooperation without an increased risk of rebellion, by virtue of an added security net. Overall, the likelihood to rebel would decrease as the Helots knew there was a greater possibility of defeat, and the payoff for *Case 2* would change in favor of the Spartiates as foreign assistance compensated for a disorganized and low-quality Spartiate army. An equilibrium of *Case* 4—when the Spartiates do not cooperate and the Helots do not rebel-was approached near the 6th century B.C, when Sparta's diplomatic prestige was at its zenith and its Helot problem well under control (Kagan, 1987). In this time period, the Lycurgan constitution was not culturally entrenched, as reflected in salient socio-economic inequalities. The poet Alkaios, writing around 600 BC, cites a certain Aristodemos from Sparta: "man is what he owns; no poor man is good or honorable (Hodkinson, 2009)." Therefore, there were no strong indications of a complete militarization that characterized Classical Sparta. This phenomenon was accompanied by the remarkably peaceful Helot situation at home.

City-State Subversions

Yet, the converse can happen as well. Sparta's notable rivals—Argos and Athens—can weaken Sparta's military strength, or collude with the Helots to overthrow the government. Argos and Arcadia came to the assistance of the Helots in the Second Messenian War. Meanwhile, Athens constructed an outpost for Helots at Pylos, which served as a reconnaissance base for military incursions towards Sparta in The Peloponnesian War (Kagan, 1987). The protracted Peloponnesian Wars from 431-404 B.C between Sparta and Athens also distracted Sparta from suppressing domestic dissatisfaction, which was reflected in short invasion campaigns of Attica—the historical region which encompasses Athens—that lasted 30 days at most. The Spartans had to tend with growing signs of revolt back at home while the bulk of its army was away, therefore it could not engage in long wars in foreign lands. In certain situations, the Helots' temerity is reflected in higher payoffs for rebelling due to greater possibility of winning a direct conflict, either because there were allies, or the Spartiate army was weakened from foreign wars.

Natural Disasters

Natural disasters such as the earthquake of 464 B.C were the proximate cause of the First Peloponnesian War, which was ignited by a Helot rebellion. According to Plutarch, the city was razed to ruins, and only 5 houses were left intact (Kouskouna & Kostas, 2004). Many of the young, prospective citizen-soldiers training in the gymnasium died after the roof collapsed. Spartiate military strength was greatly reduced, emboldening the Helots to rebel as they knew that even if the Spartiates were to cooperate, the numerical disadvantage and lack of supplies would render them militarily defunct.

Spartiate-Helot Dynamics

Additionally, the model simplifies the Spartiate-Helot dynamics. Spartiate men often took Helot mistresses, and it was likely that poorer Spartiates often labored together with their slaves to make ends meet (Hodkinson, 2009). The bitter relationship posited between the two classes might not be wholly applicable at times. The Messenian Helots were also distinguished from the Laconian Helots, who were acquired earlier than the former and had an ambiguous, possibly supportive attitude towards the Spartiates by accompanying them to battle (Chambers, 1978). More tellingly, the Helots that accompanied the Spartiates into battle could be given citizenship after completing military service. Attaining the class of *Neodamodeis*—"lately made one of the people"—these Helots are proof of social mobility existing in a society otherwise viewed to be ossified (Cartledge, 2013). A more fine-tuned study of class relations is needed for a more comprehensive model.

Conclusion

This article has demonstrated the possibility of applying mathematical analysis to social history. By modeling classes as independent players acting in self-interest, we weed out counterfactual histories. However, the model simultaneously reveals the difficulty of explaining history with math due to the large number of uncertainties and external factors. Moreover, the lack of statistical data precludes a truly convincing argument. Still, the interdisciplinary marriage of mathematics and history has proven to be rich in potential.

> JIAXIN LIU Hwa Chong Institution, Singapore

Appendix

Below is an attempt to mathematically model the strategies of the Spartiates and Helots. Table 1. Notations

Notation	Meaning	
OB	Benefit of cooperation	
OC	Cost of cooperation	
С	Spartiate casualty rates upon winning	
В	Benefit of exploitation	
δ	Multiplier for increased intensity of	
	exploitation	
R	Spartiate/Helot casualty rates upon losing	

For simplicity's sake, we do not differentiate between Spartiate and Helot casualty rates upon the scenario of losing the war. It is also justifiable to do so when we consider the tremendous decrease in Spartiate military efficacy in the case of non-cooperation and their numerical disadvantage. The net payoff of cooperation is represented by the summation of benefit and cost, which is OB - OC.

Case 1: [Rebel, Cooperate]

For Spartiates, the net payoff will be OB - OC (prerequisite for cooperation) – C (the high cost of manpower in defeating the Helots) + δB (*B* refers to the value of exploitation, while δ is the multiplier greater than 1, representing the increased degree of exploitation following a Spartiate victory).

For Helots, the net payoff will be $-\delta B - R - \delta B$ represents the increased loss of welfare following intensification of maltreatment and demands by their overlords, and *R* the immediate loss of manpower by losing the conflict (i.e. casualty rates)

Case 2: [Rebel, Don't Cooperate]

For Spartiates, the net payoff will be -B - R, where *R* stands for the immediate loss of Spartiate manpower, while *B* refers to exploitation by the Helots as roles are reversed.

For Helots, the net payoff will be *B* as they exploit their former masters.

Case 3: [Don't Rebel, Cooperate]

For Spartiates, the net payoff will be B + OB - OC. The absence of conflict precludes any immediate loss of manpower.

For Helots, the net payoff will be -B. The absence of δ also indicates the peaceful coexistence between the two classes.

Case 4: [Don't Rebel, Don't Cooperate]

For Spartiates and Helots, the net payoffs are mirror images of each other: -B and B. While this might seem to be an unlikely decision for the Helots—as they can easily obtain a greater payoff by rebelling—this can be justified from the Spartiates' perspective if the net payoff for cooperation, OB - OC, is sufficiently low.

Analysis

Table 2. Simultaneous game payoff matrix

	Helots	
	R	DR
	$\underline{OB - OC - C + \delta B}$	B + OB - OC
C	$-\delta B - R$	<u>B</u>
DC	-B-R	B
	<u>_B</u>	-B

A payoff matrix as illustrated above can help better clarify possible decisions and their payoffs. For identification purposes, Helot actions are colored blue, while Spartiate actions are black. We assume this to be a simultaneous game with zero external influence, and that both the Spartiates and Helots are unable to resolve their differences in social status, which is historically unlikely as well.

The preferable strategy for Helots under each of the two scenarios of Spartiate cooperation or non-cooperation are underlined. Concurrently, we consider the Spartiates' decisions. When Helots rebel, the Spartiates are strictly better off by cooperating if and only if:

$$OB - OC - C + \delta B > -B - R \tag{1}$$

$$OB - OC + R > -\delta B - B \tag{2}$$

$$OB - OC + R > -(\delta + 1)B \tag{3}$$

Since $\delta > 1$,

$$OB - OC + R > -2B \tag{4}$$

$$OB - OC > -R - 2B \tag{5}$$

Provided that OB - OC is greater than -R - 2B, we can conclude that Spartiates are better off by cooperating. Given the highly negative value of -R - 2B, we can reasonably estimate that this assumption holds. After all, the undeveloped economy of Sparta greatly reduced the degree of liquidity and monetization (Hodkinson, 2009). Assets could not be flexibly transferred or converted for greater utility purposes. Therefore, *OC* does not seem significant. The best strategy for the Spartiates when the Helots rebel will be to cooperate, which is underlined in the table above.

When Helots do not choose to rebel, whether it is more advantageous to cooperate or not cooperate is contingent on the value of OB - OC. If OB - OC > 0, the Nash equilibrium of *Case* 3 is achieved, as attested by the underlining of both Spartiate and Helot payoffs in this square. This is because B + OB - OC > B. It is a scenario whereby both Helots and Spartiates refuse to deviate. If OB - OC < 0, the preferable strategy for Spartiates is to not cooperate when the Helots do not rebel, as attested by the underlining of *B* in the square below. In this scenario, there is no Nash Equilibrium, since there is no outcome that is in both players' self-interest to achieve. Instead, in accordance to their self-interest, their preferred strategies span the entire range of the 4 possible outcomes. This indicates some extent of fragility and uncertainty to the social structure of Sparta.

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